Where to take a rugby place kick

The sport of rugby is a lot like American Football, but when you score a touchdown (they call it a “try”) you take the extra point kick (they call it a “conversion”) from a place in the field in line with the place the try was scored. So if the try is scored close to the sideline, the kick is taken from somewhere close to the sideline. If it is scored in the middle of the field, the kick can be taken from the middle of the field (Figure. 1).

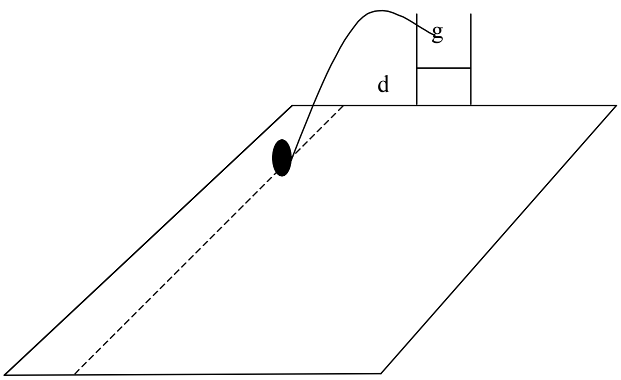

Figure 1: Rugby extra point kick is taken from a position on the field in line with the location of the touchdown

If the goalposts are width g, and the try is scored distance d outside the left post, what is the best place to take the kick?

This will turn out to be a problem within the following topic areas:

- Calculus optimization
- Conic curves

Finding the “best” place is usually mapped into maximizing or minimizing something. But what should we be maximizing or minimizing?

What about distance, where would the closest point to the goal posts on the permitted line be?

Would this be a good place to kick from? If not, why not?

So if not distance, what else could be optimized? How about angle? Presumably it would be best from a point of view of accuracy if the angle between the sightline to the near post and the sightline to the far post were as large as possible.

Use Geometry Expressions to get a formula for the size of this angle in terms of the width of the goals g, the distance between the near post and the location where the try was scored d, and the distance x from the goal line (Figure 2).

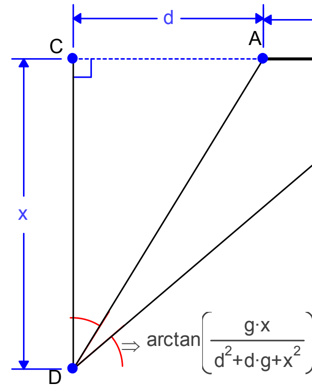

Figure 2: Angle made by the goalposts at the point of the kick

Let’s assume that d is positive (that is, the try was not scored underneath the goal posts.) What can you say about angle BAD? What can you say about angle ADB?

Have a look at a graph of the arctan() function. What is its domain? Is it monotonic over that domain?

Is it true that the maximum value of the angle comes at the maximum value of the argument of the arctangent?

Use calculus to find the value of x which maximizes this quantity.

When you have found your solution, you can plug it back into the diagram, replacing x by the solved value:

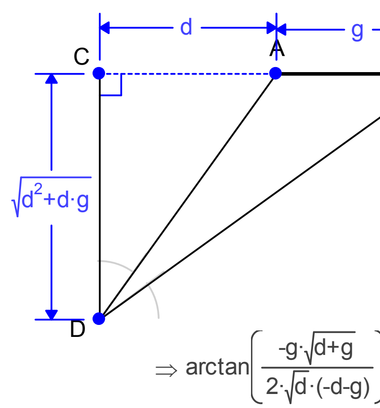

Figure 3: Solution configuration

Try dragging the point D (You may need to lock g, to keep the width of the goalposts fixed!).

We have created an algebraic answer to the problem, and we have a geometric diagram of a single solution. How can we create a diagram which shows the solutions for all values of d?

Create the locus of the solution point as d varies.

We will want to look at the equation of the locus in due course, we should fix our drawing in the coordinate plane first. One way to do this would be to remove the length constraint on the goal and instead specify the coordinates of each goal post, (Figure 4).

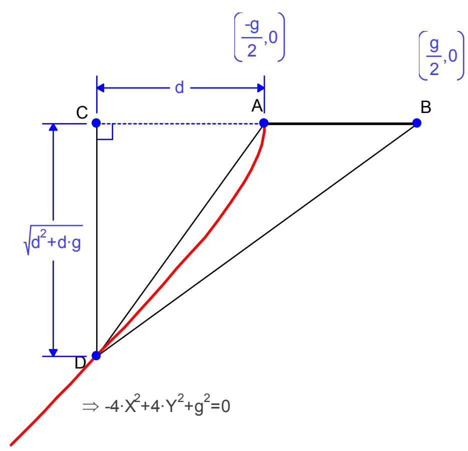

Figure 4: Locus of optimal kick points is a hyperbola

What kind of curve is this?

Advice for a rugby kicker

We could tell a rugby kicker to place his ball on a specific curve, however, it is unlikely that in practice he could follow that advice. Is there some way we could approximate the curve and give the rugby kicker a rule of thumb to work with?

Can you find an asymptote to the curve and plot it in Geometry Expressions?

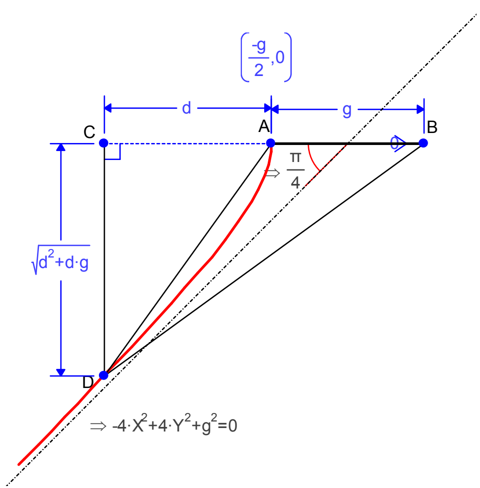

Figure 5: The hyperbola asymptote is the line Y=X

Is the asymptote a good approximation of the curve?

What practical advice would you give to a rugby kicker?

Further Questions

What about the situation where the try is scored between the posts?

What is the maximum value of the angle, and where is this attained?

Would this be a good place to kick from?

What factors do you think would influence the placing of the kick in this situation?

Notes

The derivative of the argument of the arctan function is:

$$\frac{gx^2 - gd^2 - g^2 d}{(d^2 + dg + x^2)^2}$$

Solving for x gives:

$$x = \sqrt{d^2 + dg}$$