

101 Symbolic Geometry Examples

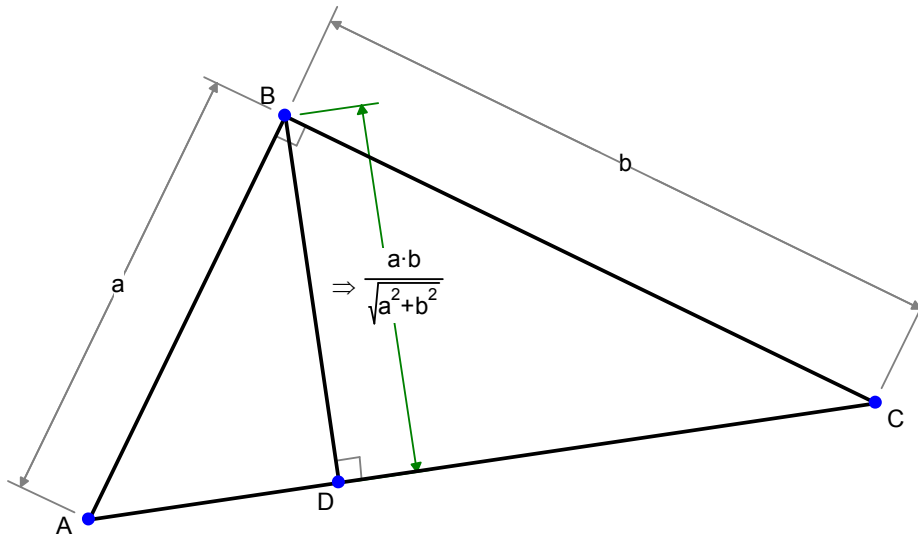
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Introduction

Geometry Expressions is a software application which automatically generates algebraic expressions from geometric figures. For example in the diagram below, the user has specified that the triangle is right and has short sides length a and b . The system has calculated an expression for the length of the altitude:



This book comprises 101 examples of the use of Geometry Expressions in a variety of settings. In some cases, we give a simple model with little explanation. In other cases, there is some exposition backed up by Geometry Expressions models. Some examples make use of an algebra system in addition to Geometry Expressions, while many use Geometry Expressions stand-alone. Together, we hope, they give an indication of what the system can do, and provide a starting point for the reader to pursue his own discoveries.

Example 1: Median & Angle Bisector of a Right Triangle

In the introduction, we looked at the altitude of a right angled triangle. Here we examine the lengths of the median and the angle bisector. Can you prove these results geometrically?

