Efficiency of Circular Solar Cookers with Varying Incoming Angles

Ariel Chen

Introduction:

Parabolas are popular curves used for solar concentrators because of their ability to reflect incoming light parallel to the central axis to a single focal point. However, parabolic curves are inefficient when the light is not parallel. This paper investigates another possible curve, the circle, and explores the efficiency of the circular solar concentrator at incoming angles not parallel to the central axis with mathematical models created by Geometry Expressions, a constraint-based symbolic geometry system.

The Model:

The main components of a solar concentrator are the concentrator and the target. In our model, the light enters the concentrator horizontally. The boundaries of the concentrator are marked off by the pink dots that are on both sides of the reflector. The boundaries are constrained so the aperture length of the solar concentrator is always 1 unit. All other variables are determined as a ratio to the aperture length. The target of the concentrator is represented by the smaller the circle with radius *r*. The incoming ray is represented by the red line with an incoming angle θ (i.e. 90^{o} represents when the concentrator is accurately pointing at the sun). The orange line is the reflected ray which is a reflection of the incoming ray off the tangent at the incidence point. *f* is the radius of the larger circle. Changing *f* changes the radius of curvature within the boundaries of the solar concentrator, and *d* is the distance of the center of the target to the center of the large circle.

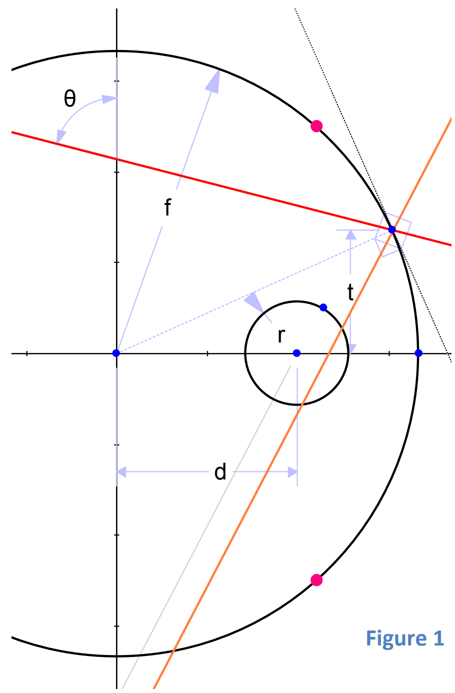

Figure 1

Problem: Target Placement

Unlike the parabola, the circle does not have a single point that all reflected rays converge to, so there is no obvious placement for the target. The circle does, however, produce a nephroid-shaped caustic with a cusp at f/2 from the reflected rays.

The two possible positions for target placement would be with the tip of the target located at the cusp of the caustic (Figure 2) or with the target tangent to the caustic (Figure 3).

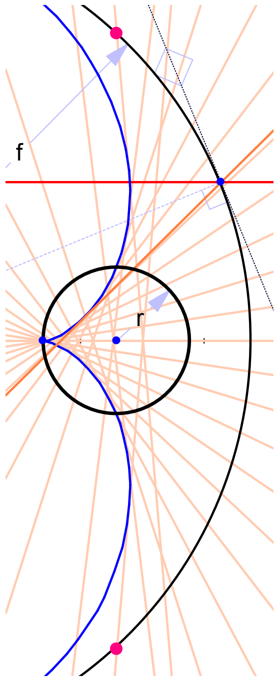

Figure 2

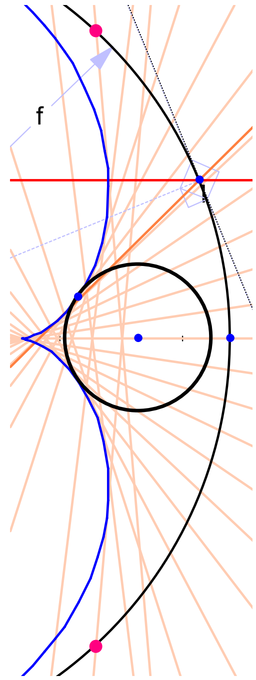

Figure 3

These two special conditions will be investigated later.

Using Geometry Expressions, the distance between the reflected ray and the center of the target in the generic model (Figure 1) is:

$$\left| -\sin(\theta)\, t + \frac{2\sin(\theta)\sqrt{f^2 - t^2}\, t\, d}{f^2} + \cos(\theta)\, d - \frac{2\cos(\theta)\, t^2\, d}{f^2} - \cos(\theta)\sqrt{f^2 - t^2} \right|$$

Using this formula, a program was written in Maple that projects a sequence of rays at a constant interval from (*t*=-0.5) to (t=0.5) (or the boundary of the reflector). It calculates whether the distance is greater or less than the specified radius. If it is less than the radius then the specific ray hit the target, and if it is greater than the radius then the specific ray missed. Then the number of hits and misses were tabulated and converted into percent efficiency. Percent efficiency is the percent of light rays within a given boundary that reflect through the target.

To graph the different rates at which percent efficiency declines as the incoming angle decreases with different target positions, *f* and *r* had to be specified to a specific value. Changing *r* has no analytical significance because it is obvious that as *r* increases the angle tolerance and percent efficiency will increase; however, increasing *r* decreases the concentration ratio which decreases the power of the solar concentrator. *r* was set at the value 0.12 for this specific investigation.

Changing *f* however changes the radius of curvature of the curve. From a previous investigation with parabolic reflectors, the optimal angle tolerance occurs when the center of the target is level with the boundaries of the reflector. For comparison, we use the same geometric configuration with the center of the target level with the boundaries for the circle. Using the Geometry Expressions model where the target is located right beneath the cusp (Figure 2), the *f* value where the center of the target is level with the boundaries of circular concentrator is when

$$f = \frac{2}{3}r + \frac{1}{3}\sqrt{16r^2 + 3}$$

When $r = 0.12$, then $f = 0.6791104517$.

Figure 4 graphs the percent efficiency as the incoming angle departs from the perpendicular for $r = 0.12$ and $f= 0.6791104517$.

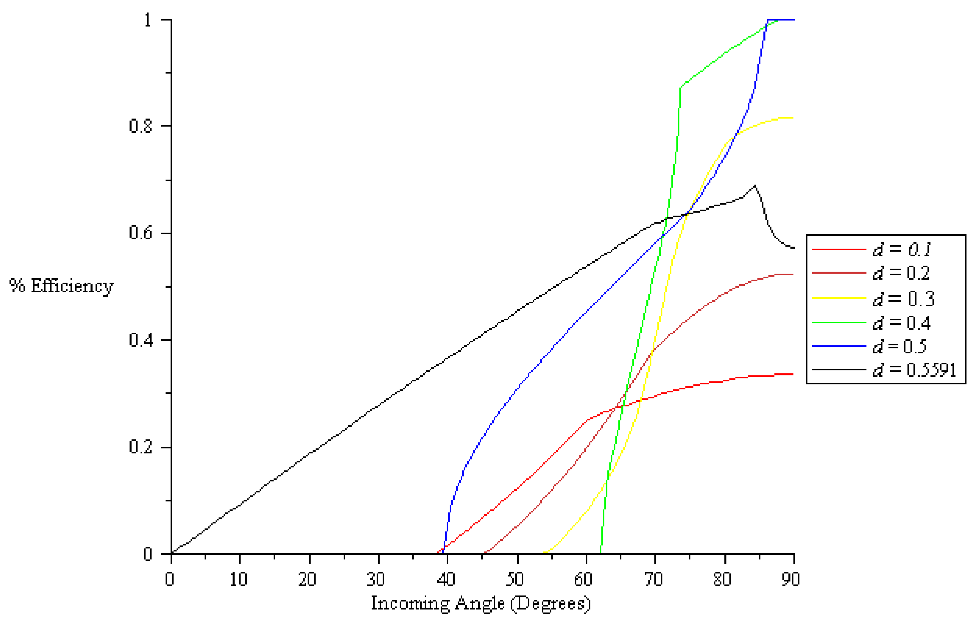

Figure 4

Each curve represents a different value of *d* as specified from Figure 1. This graph shows how the efficiency or percent of light within boundaries that reflect to the target decreases as the incoming angle decreases. Although it does not represent all possible values of *d* it shows how the rate of decrease in efficiency changes as the target is moved away from the center of the large circle. When *d* is from 0.1 – 0.3, the reflector never reaches 100% efficiency, meaning that it is impossible for all the light that reflects within the boundaries of the solar cooker to reflect to the target with that given *d* value. It is only possible to reach higher rates of efficiency when *d* is around 0.4 and 0.5, although the lower values of *d* are able to keep a small amount of light reflected to the target at smaller incoming angles. The most suitable value of *d* for a concentrator would be when $d = 0.5$. When the target is located at that position, it is able to be fully efficient for a small range of incoming angles, and then still have some efficiency at smaller incoming angles.

The two special cases from Figure 2 and Figure 3 have specific values of *r*. When the edge of the target is at the tip of the cusp (Figure 1) then *d* = f/2 + *r*. So *d* in Figure 2 with *r* = 0.12 and *f* = 0.6791104517 is .4595552259.

Using Geometry Expressions in conjunction with Maple, when the target is tangent to the cusp then

$$r = \frac{1}{2}\left(\left(f^2 - \left(\frac{1}{6}\left(108\, r f^2 - 64\, r^3 + 12\sqrt{81\, r^2 f^4 - 96\, r^4 f^2}\right)^{1/3} + \frac{8}{3}\,\frac{r^2}{\left(108\, r f^2 - 64\, r^3 + 12\sqrt{81\, r^2 f^4 - 96\, r^4 f^2}\right)^{1/3}} - \frac{2}{3}\, r\right)^2\right)^{1/2} f^2\right) \Bigg/ \left(f^2 - 2\left(\frac{1}{6}\left(108\, r f^2 - 64\, r^3 + 12\sqrt{81\, r^2 f^4 - 96\, r^4 f^2}\right)^{1/3} + \frac{8}{3}\,\frac{r^2}{\left(108\, r f^2 - 64\, r^3 + 12\sqrt{81\, r^2 f^4 - 96\, r^4 f^2}\right)^{1/3}} - \frac{2}{3}\, r\right)^2\right)$$

So *d* in Figure 3 with *r* = 0.12 and *f* = 0.6791104517 is 0.5292912430.

The results were graphed with the special values of *d* for *d* = 0.5.

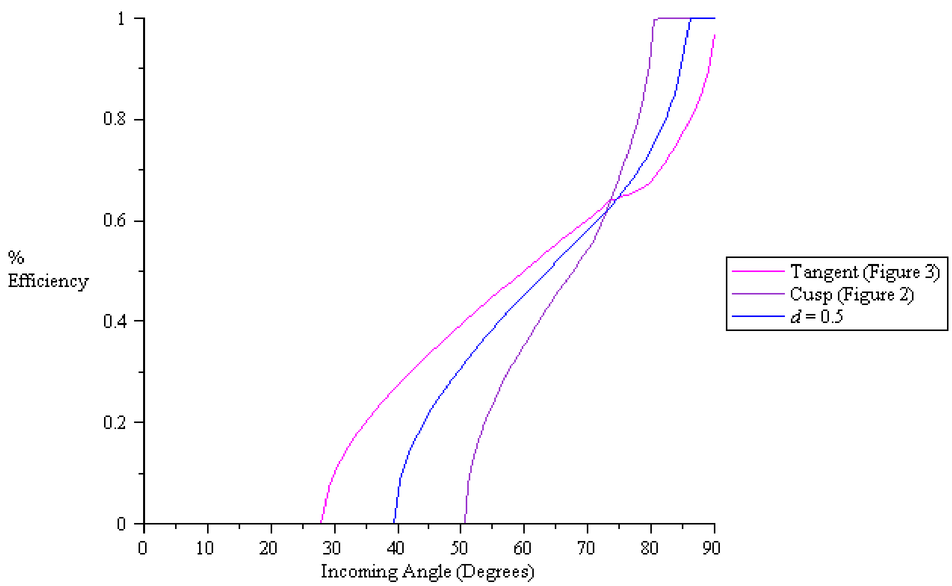

Figure 5

Problem: Radius of Curvature by *f*

In the previous section, how efficiency changed as the target moved away from the center of the circle was observed. For this section, the target will be held at the cusp of the caustic while the radius of curvature is changed. Because the boundaries of the solar concentrator are held at a constant distance of 1, changing *f* changes the curvature of the concentrator shown below:

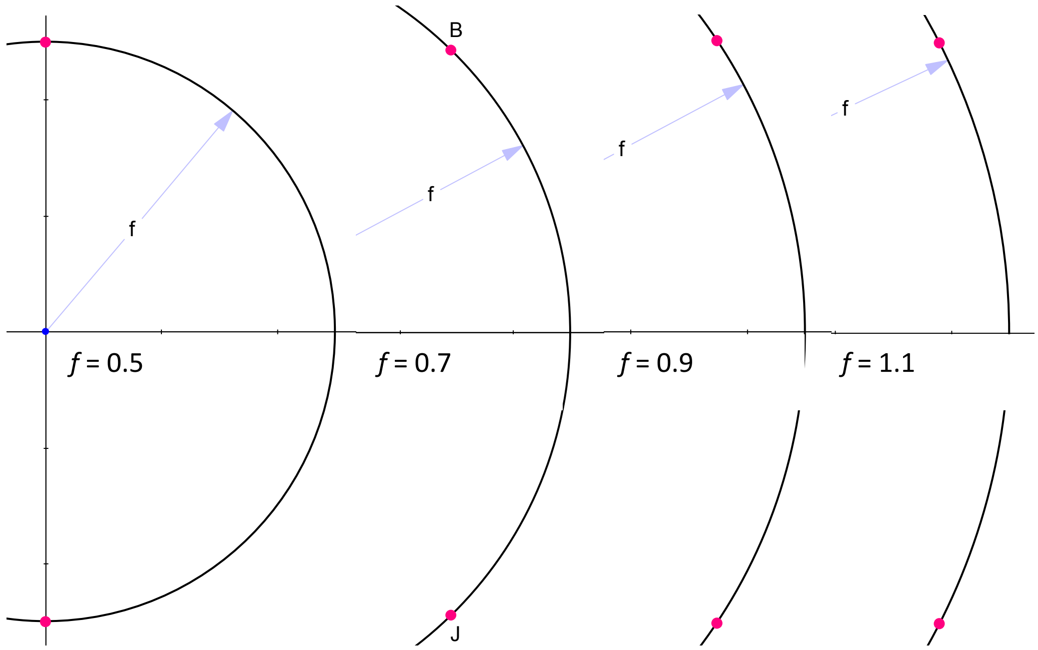

Figure 6

Using Geometry Expressions, the distance between a reflected ray and the center of the pot is:

$$\frac{1}{2}\left|\frac{1}{f^2}\left(-2\sin\left(\frac{1}{180}\theta\pi\right)t f^2+2\sin\left(\frac{1}{180}\theta\pi\right)\sqrt{f^2-t^2}\,t f\right.\right.$$
$$+4\sin\left(\frac{1}{180}\theta\pi\right)\sqrt{f^2-t^2}\,t r+\cos\left(\frac{1}{180}\theta\pi\right)\left(-f^3\right.$$
$$\left.\left.\left.-2 r f^2+2t^2 f+4t^2 r+2\sqrt{f^2-t^2}\,f^2\right)\right)\right|$$

Using the same ray tracing technique describe before, the percent efficiency as the incoming angle departs from the perpendicular. Figure 7 shows how the rate of decreasing efficiency changes as *f* changes. The radius of the target was held constant at 0.12.

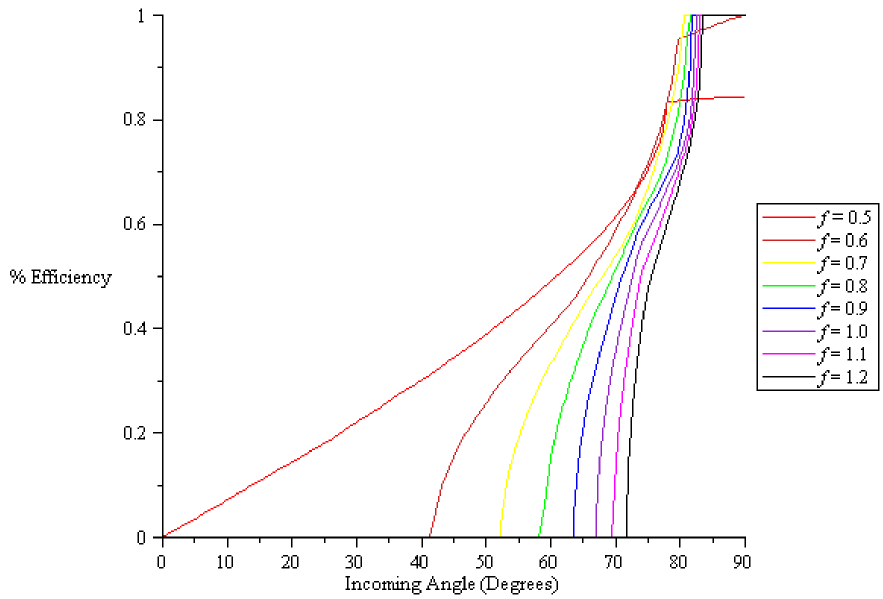

Figure 7

The solar concentrator has the highest angle tolerance when the curve is a semi-circle (*f* = 0.5). However, it never reaches full efficiency. The flatter the curve of the reflector (larger values of *f)* the faster efficiency decreases as the incoming angle decreases.

Comparison of the Parabolic and Circular Curve Efficiency

Knowing how percent efficiency decreases as the incoming angle decreases for circular reflectors, we can now examine our original question: does a circular curve reflect light better at incoming angles with a greater departure from the perpendicular?

The parabola model is shown in Figure 8. In this model *f* is the focal height of the parabola, and the center of the pot is located at the focal point. The aperture length of the parabola is also held constant at 1.

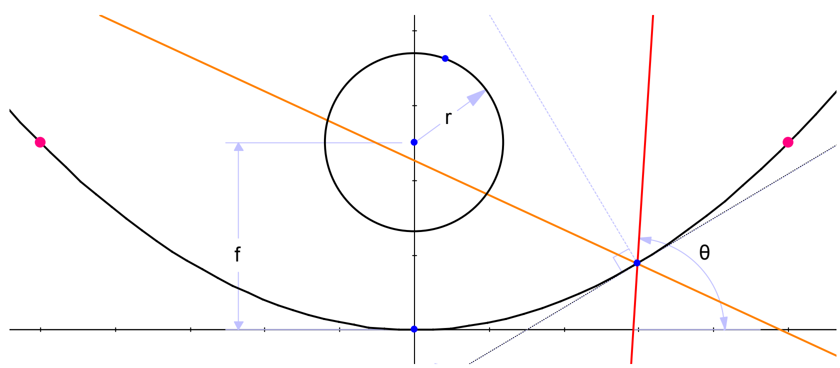

Figure 8

From previous investigations, the parabola is always the most angle tolerant when the focal height is level with the boundaries of the reflector. Using this optimal angle tolerance combination with $r = 0.12$, the decrease in percent efficiency as incoming angle decreased for the parabola was compared with the circle in Figure 9. The value of f used for the circle was when the center of the circle located at the tip of the cusp (Figure 2) was level with the boundaries.

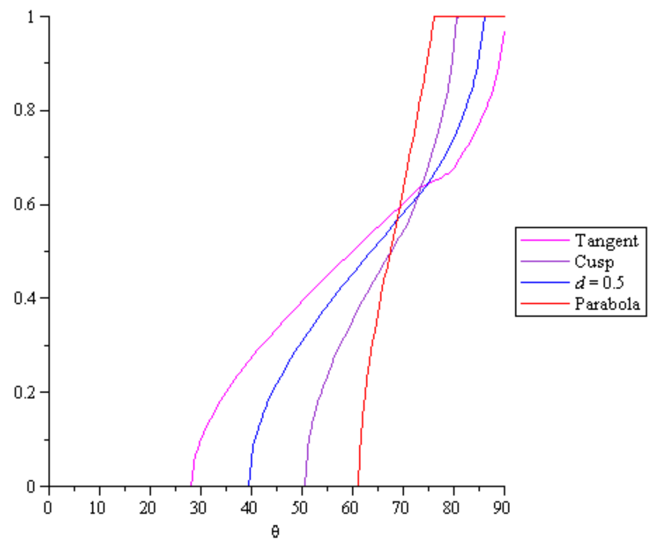

Figure 9

Conclusion:

Although the parabola is more accurate than any one of the different circular solar concentrators, the circular concentrators are far more angle tolerant. There is a trade-off between angle tolerance and full efficiency. The wider the range of incoming angles that are fully efficient, the less angle tolerant it is, and the percent efficiency decreases at a faster rate after the critical angle.

The crossover area where the circular concentrators are more efficient than the parabolic concentrator is at approximately when the incoming angle is 60^0 with 60% efficiency. If we are prepared to accept less than 60% efficiency, then the circular concentrator would be more beneficial because it is able to concentrate light at slanted incoming angles. Also, a concentrator that would need a range of angles larger than 60^o, the equivalent of four hours, would be more efficient with a circular curve rather than a parabolic curve.

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