Learning Calculus With Geometry Expressions™

by L. Van Warren

$$\pi$$

Introduction

Learning Calculus with Geometry Expressions™

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Organization and Examples

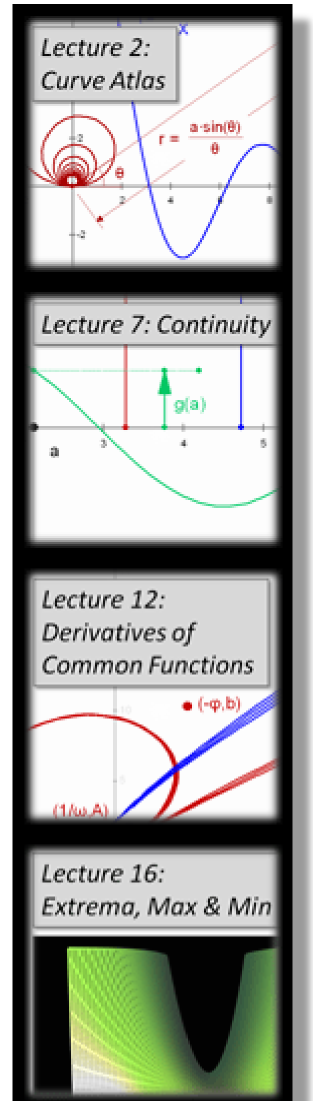
Learning Calculus consists of twenty-six lectures grouped in six chapters. Each chapter contains four or five lectures in pdf format.

Each lecture refers to Geometry Expressions example files suffixed with a *.gx* extension. These are read-only, but you may edit them if you wish by using the **File/Save As** command and renaming them.

The lectures also use the free computer algebra system, [wxMaxima]™. These examples are suffixed with a *.wxm* filename extension.

The text also refers to a few Excel spreadsheet files, supplied in the new and old versions, *.xls* and *.xlsx*.

In one lecture on reaction rates, *.c3xml* chemistry files are included, these can be read using CambridgeSoft chemistry software, but this is not necessary for understanding the mathematics.



How To Use This Book

"Line upon line, precept upon precept."

This is a fun book.
When you tire, take a break and return.
Work the examples, one at a time,
checking them off when you're done.

Early examples feature "Gray Box Help".
Follow the numbered steps, one at a time until you understand.

Gray Box Help
1) Continue reading.
2) When finished, turn page.

At first, we will construct the examples for you.
Later, you will construct them for yourself.

Bit by bit, line by line…

Stalking the Wild Asparagus:

- Nine percent of respondents report that Asparagus reminds them of the integral sign.

In the last lecture we examined antiderivatives – shown as equivalent to indefinite integrals.

This lecture is about finding the area under a curve. We will do this by using the definite integral, we will start by reviewing some important results.

$$\int_1^2 x\,dx = \left.\frac{x^2}{2}\right|_1^2 = \frac{2^2}{2} - \frac{1^2}{2} = 2 - \frac{1}{2} = \frac{3}{2}$$

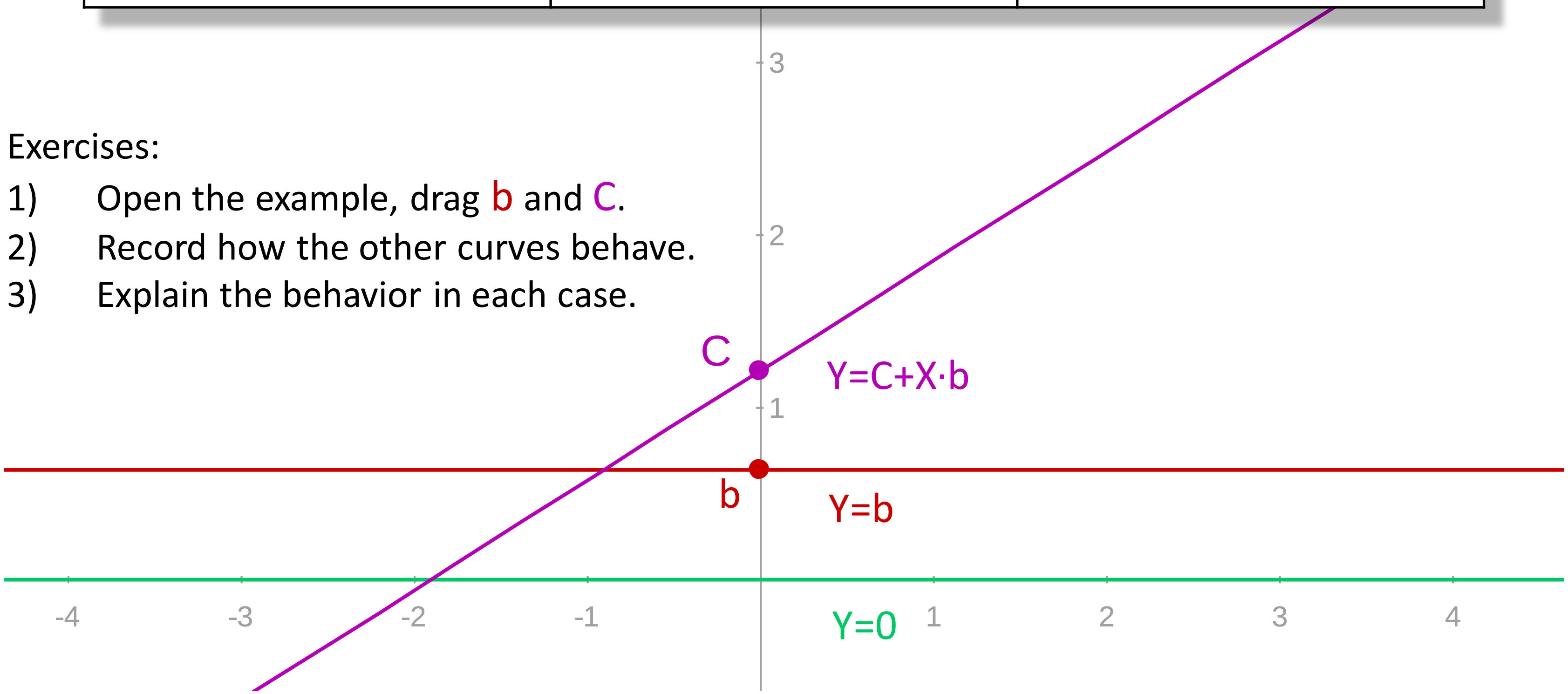
Lecture20-IntegralOfConstant.gx

Derivatives & Integrals: Constant Case

Derivative	Function	Integral
$\frac{d}{dx} b = 0$	b	$\int b\,dx = bx + C$

Exercises:

1) Open the example, drag b and C.
2) Record how the other curves behave.
3) Explain the behavior in each case.



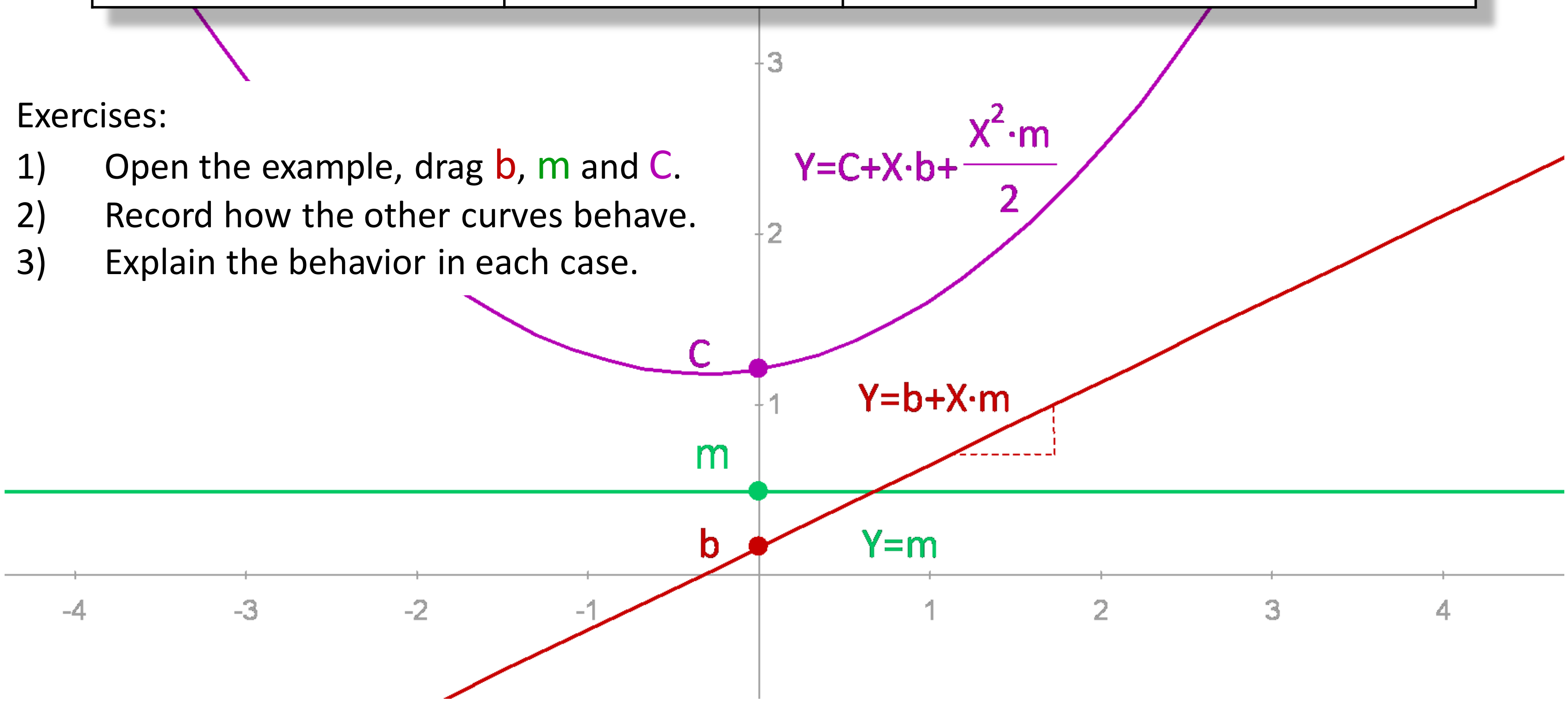
Lecture20-IntegralOfLinear.gx

Derivatives & Integrals: Linear Case

Derivative	Function	Integral
$\frac{d}{dx}(mx+b)=m$	$mx+b$	$\int (mx+b)dx=\frac{m}{2}x^2+bx+C$

Exercises:

1) Open the example, drag b, m and C.
2) Record how the other curves behave.
3) Explain the behavior in each case.



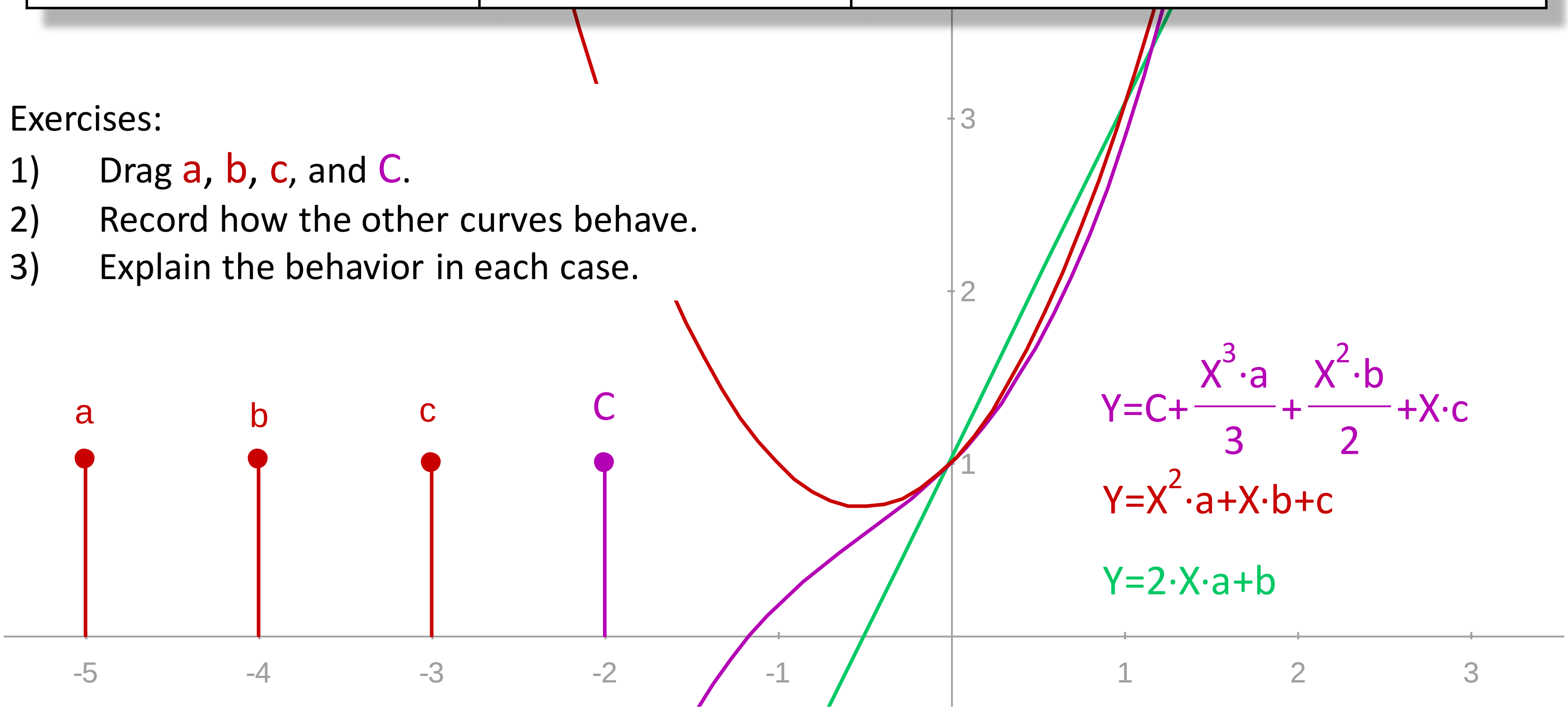
Lecture20-IntegralOfQuadratic.gx

Derivatives & Integrals: Quadratic Case

Derivative	Function	Integral
$2ax+b$	ax^2+bx+c	$\frac{a}{3}x^3+\frac{b}{2}x^2+cx+C$

Exercises:

1) Drag a, b, c, and C.
2) Record how the other curves behave.
3) Explain the behavior in each case.



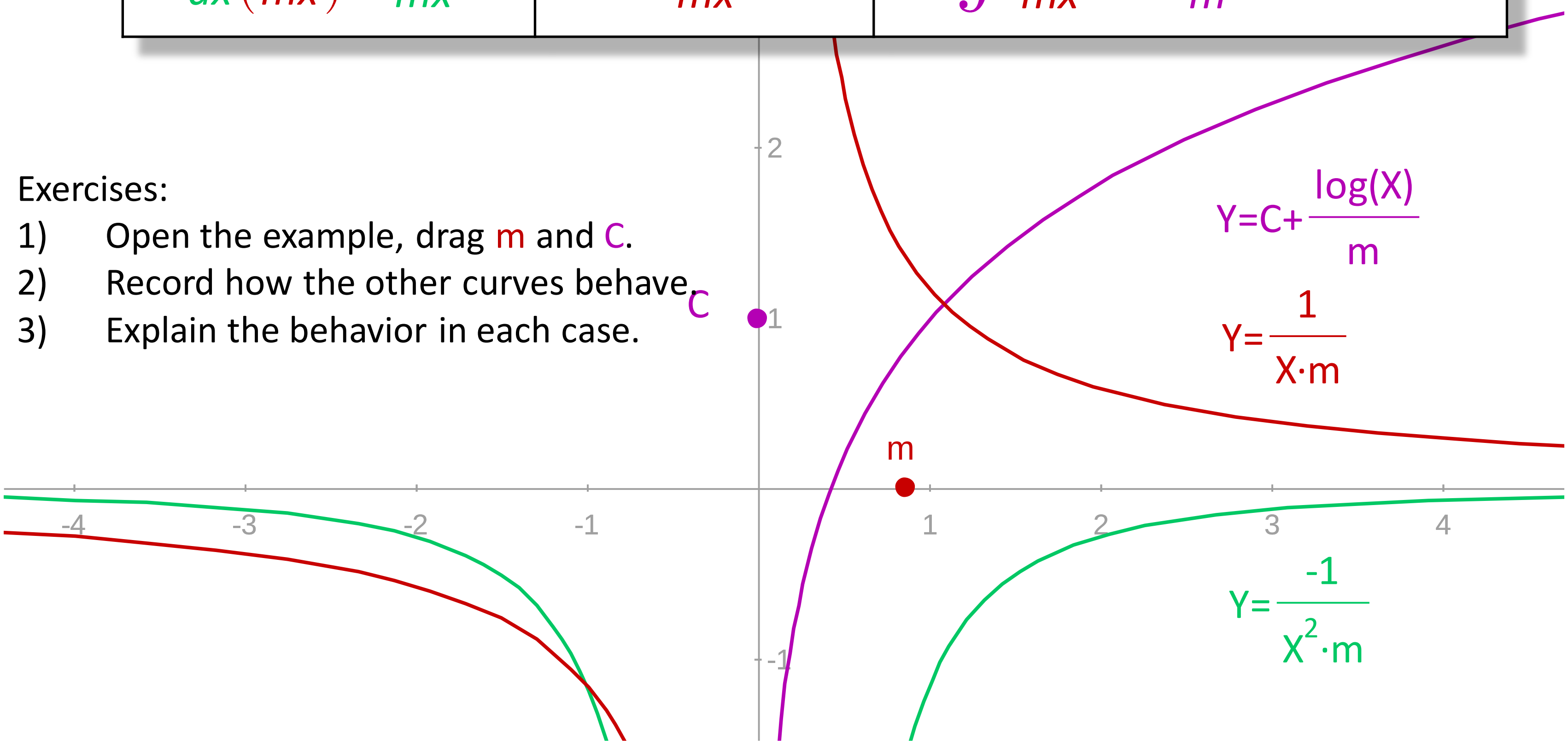
Lecture20-IntegralOfInverse.gx

Derivatives & Integrals: Inverse Case

Derivative	Function	Integral
$\frac{d}{dx}\left(\frac{1}{mx}\right)=\frac{-1}{mx^2}$	$\frac{1}{mx}$	$\int\frac{1}{mx}dx=\frac{1}{m}log(x)+C$

Exercises:

1) Open the example, drag m and C.
2) Record how the other curves behave.
3) Explain the behavior in each case.



Derivatives & Integrals: Other Cases

Derivative	Function	Integral
nax^{n-1}	ax^n	$\frac{a}{n+1}x^{n+1}+C$
$A\cos(x)$	$A\sin(x)$	$-A\cos(x)+C$
$A\omega\cos(\omega t+\phi)$	$A\sin(\omega t+\phi)$	$-\frac{A}{\omega}\cos(\omega t+\phi)+C$
e^x	e^x	e^x
$\frac{1}{x}$	$\ln(x)$	$x\ln(x)-x$

Exercise: Create a Geometry Expressions™ example for each case above.